LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034

B.Sc. DEGREE EXAMINATION – **MATHEMATICS**

FIFTH SEMESTER – **APRIL 2023**

UMT 5601 – GRAPH THEORY

Date: 15-05-2023 Dept. No. Max. : 100 Marks

Time: 01:00 PM - 04:00 PM

	PART – A	**(10 x 2 = 10 Marks)**
Q. No.	**Answer ALL questions**	
1	Prove that the number of vertices of odd degree in a graph is always even.	
2	Define connected graph.	
3	Draw complete graphs on 5 and 6 vertices.	
4	Given an example of Hamiltonian graph, but not Euler graph.	
5	Draw all possible non-isomorphic trees on six vertices.	
6	Define center of a graph and find the centers of a circuit with 4 vertices.	
7	Define a fundamental cut-set.	
8	Differentiate vertex connectivity and edge connectivity.	
9	Define a planar graph.	
10	What is chromatic number?	
	PART – B	**(5 x 8 = 40 Marks)**
Answer any FIVE questions		
11	Prove that a graph G is disconnected if and only if its vertex set V can be partitioned into two nonempty disjoint subsets V_1 and V_2 such that there exists no edge in G whose one end vertex is in V_1 and the other end in V_2.	
12	Prove that a connected graph G is Euler if and only if it can be decomposed into circuits.	
13	Prove that a tree with n vertices has $(n-1)$ edges.	
14	With respect to a given spanning tree T, prove that a chord c_i determines a fundamental circuit Γ which occurs in every fundamental cut-set associated with the branches in T and in no others.	
15	Prove that the complete graph of five vertices is nonplanar.	
16	Prove that every circuit has an even number of edges in common with any cut-set.	
17	Explain the following graph operations with examples: intersection, union and ring sum.	
18	Show that every tree with two or more vertices is 2-chromatic.	

PART – C

Answer any TWO questions. **(2 X 20 = 40 Marks)**

19	(a)	Prove that a connected graph G is an Euler graph iff all vertices of G are of even degree.	(10)
	(b)	Show that a simple graph with n vertices and k components can have at most $\frac{(n-k)(n-k+1)}{2}$ edges.	(10)
20	(a)	Prove that every tree has either one or two centers.	(10)
	(b)	If n is an odd number and $n \geq 3$, prove that in a complete graph with n vertices there are $(n-1)/2$ edge-disjoint Hamiltonian circuits.	(10)
21	(a)	Prove that the ring sum of any two cut-sets in a graph is either a third cut-set or an edge disjoint union of cut-sets.	(10)
	(b)	Prove that a connected planar graph with n vertices and e edges has $e-n+2$ regions.	(10)
22	(a)	Explain the digraph and write down the advantages and disadvantages over an undirected graph.	(10)
	(b)	Show that a graph with at least one edge is 2-chromatic if and only if it has no circuits of odd length.	(10)

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